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LETTER TO THE EDITOR

Path integrals for quantum algebras and the classical limit†

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Abstract. Coherent states path integral formalism for the simplest quantum algebras, q-oscillator, $SU_q(2)$ and $SU_q(1,1)$ is introduced. In the classical limit, canonical structure is derived with a modified symplectic and Riemannian metric. Non-constant deformation induced curvature for the phase spaces is obtained.

Aiming for a better understanding of the notion of quantum groups and algebras [1–3], it is desirable to try to elucidate the geometrical properties of these structures. The analogous properties of Lie groups emerge from the study of these groups as transformation groups in certain spaces. The symplectic and Riemannian properties of classical groups are well known and the purpose of this letter is to attempt to ask similar questions when q-deformation is present. Confronted with the vast generality of such a problem we have chosen to study three of the simplest quantum algebras, namely the deformed Weyl–Heisenberg algebra (q-WH) [4, 5] the $SU_q(2)$ and $SU_q(1,1)$ algebras. To this end we will employ the tools of path integrals and coherent states, adapted to the q-deformed situation.

Usual coherent states (CS) [6, 7] for simple Lie groups are widely used in the path integral formalism especially for Hamiltonians which are elements of the corresponding Lie algebra of the groups. The same formalism also provides in the classical limit, regarded as the case where the Planck constant is taken to be small compared to the action, the canonical equations of motion in the phase spaces, which for the groups we are concerned with here are the coset spaces $WH/U(1) \approx \mathbb{R}^2$, $SU(2)/U(1) \approx S^2$ and $SU(1,1)/U(1) \approx S^{1,1}$. Obviously in the cases of SU(2) and SU(1, 1) groups the ensuing coset spaces, i.e. sphere and hyperboloid are generalizations of the usual plane phase space of the harmonic oscillator and this gives rise to a Kähler manifold structure with a modified canonical symplectic 2-form [8].

The deformed CS [5, 9], which will be used here to build up the propagator, satisfy the completeness relation and are obtained by acting on some lowest weight with a displacement operator which involves either the deformed, or alternatively the ordinary, exponential [9, 10]. This second possibility, as will be seen in the following, has some important technical merits in the construction of path integrals and in the study of the eigenvalue problem of the q-CS.

A brief summary of our results is as follows: in a very similar manner to the changes of the geometrical structure of the harmonic oscillator phase space occurring when we pass

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to the SU(2) and SU(1, 1) phase spaces, the q-deformation of the above algebras modifies their corresponding phase spaces, as is shown by evaluating the symplectic and Riemann metrics. Also the computation of the curvature scalar reveals that a q-deformation induces a non-constant curvature in each of the above phase spaces.

Deformed coherent states for the q-WH algebra are defined by ($\alpha \in \mathbb{C}$)

$$|\alpha\rangle_q = \mathrm{e}_q^{\alpha a_q^+}|0\rangle = \mathrm{e}^{\alpha T_a^+}|0\rangle = \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{[n]!}}\,|n\rangle \tag{1}$$

where the q-oscillator commutation relations are

$$a_q a_q^+ - a_q^+ a_q = [N+1] - [N] \qquad [N, a_q^+] = a_q^+ \qquad [N, a_q] = -a_q \tag{2}$$

while

$$T_a^+ = a_q^+ \frac{(N+1)}{[N+1]} \qquad \text{and} \qquad T_a^- = \frac{(N+1)}{[N+1]} a_q \tag{3}$$

and the following symbols are used ($q = \mathrm{e}^\gamma$):

$$[n] = \frac{q^n - q^n}{q - q^{-1}} \qquad \text{and} \qquad [n]! = [1][2]\cdots[n]$$

with the q-exponential function $\mathrm{e}_q^x = \sum_{n=0}^{\infty} x^n/[n]!$ [11]. The normalized states $|\alpha)_q = (1/\sqrt{{}_q\langle\alpha|\alpha\rangle_q})|\alpha\rangle_q$ with ${}_q\langle\alpha|\alpha\rangle_q = \mathrm{e}_q^{|\alpha|^2}$ are eigenstates of the annihilation operator $a_q|\alpha)_q = \alpha|\alpha)_q$ and satisfy the completeness relation [12–14]

$$\mathbf{1} = \int |\alpha\rangle_q \,\mathrm{d}\mu_q(\alpha)_q\,\langle\alpha| \qquad \text{with} \qquad \mathrm{d}\mu_q(\alpha) = \frac{\mathrm{d}_q^2\alpha}{{}_q\langle\alpha|\alpha\rangle_q} \tag{4}$$

where the integral is regarded as the Jackson's q-integral [15]. These q-CS are minimum-uncertainty states in the sense that they minimize the $[q_q, p_q]$ commutator

$$\Delta q_q \Delta p_q = \tfrac{1}{2}|{}_q\langle\alpha|[q_q, p_a]|\alpha\rangle_q|$$

where $a_q = (1/\sqrt{2})(q_q + \mathrm{i}p_q)$ and $a_q^+ = (1/\sqrt{2})(q_q - \mathrm{i}p_a)$.

The deformed CS for the $\mathrm{SU}_q(2)$ algebra, related to representations characterized by $j = 1/2, 1, 3/2, \ldots$ are defined by ($z \in \mathbb{C}$)

$$|z\rangle = \mathrm{e}_q^{zJ_q^+}|-j\rangle = \mathrm{e}^{zT_J^+}|-j\rangle = \sum_{m=-j}^{j} \begin{bmatrix} 2j \\ j+m \end{bmatrix}_q z^{j+m}|m\rangle \tag{5}$$

where the q-binomal is defined as

$$\left[{a \atop b}\right]_q = \frac{[a]!}{[b]![a-b]!}.$$

The generators involved in the definition satisfy the commutation relations

$$[J_q^3, J_q^\pm] = \pm J_q^\pm \qquad [J_q^+, J_q^-] = [2J_q^3] \tag{6}$$

while

$$T_J^+ = J_q^+ \frac{(J_q^3 + j + 1)}{[J_q^3 + j + 1]} \qquad \text{and} \qquad T_J^- = \frac{(J_q^3 + j + 1)}{[J_q^3 + j + 1]} J_q^- . \tag{7}$$

The factor $(1 + |z|^2)_q^{2j} \equiv {}_q(z|z)_q$ normalizes the states, $|z\rangle_q = (1/\sqrt{{}_q(z|z)_q})|z)_q$ and using the general formula

$$(x + y)_q^n \equiv \sum_{m=0}^{n} \begin{bmatrix} n \\ m \end{bmatrix}_q x^{n-m} y^m = \prod_{k=1}^{n} (x + q^{n-2k+1} y)$$

derived with the help of $[2m + 1] = \sum_{\ell=-m}^{m} q^{2\ell}$, is written as

$${}_q(z|z)_q = \sum_{m=-j}^{j} \begin{bmatrix} 2j \\ m + j \end{bmatrix}_q |z|^m = \prod_{k=1}^{2j} (1 + q^{2j-2k+1} |z|^2).$$

The normalized q-CS are complete with a resolution of unity

$$\mathbf{1} = \int |z\rangle_q \, \mathrm{d}\mu_q(z)_q \, \langle z| \qquad \text{with} \qquad \mathrm{d}\mu_q(z) = \frac{[2j + 1]}{{}_q(z|z)_q^2} \mathrm{d}_q^2 z$$

where again the Jackson's q-integral is used.

It is also interesting that the q-CS satisfy the eigenvalue problem

$$(J_q^- + (q^j + q^{-j}) z [J_q^3] - z^2 J_q^+) |z\rangle_q = 0 \tag{8}$$

which upon taking the zero deformation limit reduces to its analogous $q = 1$ equation which serves as a definition, up to a phase factor, for the SU(2) coherent state [7]. For future use we also record the formula

$$J_q^{\pm} |z\rangle_q = z^{\mp 1} [j \pm J_q^3] |z\rangle_q . \tag{9}$$

The coherent states related to the quantum SU(1, 1)

$$[K_q^3, K_q^{\pm}] = \pm K_q^{\pm} \qquad [K_q^+, K_q^-] = -[2K_q^3] \tag{10}$$

and associated with the discrete representations characterized by $k = 1, 3/2, 2, 5/2, \ldots$ are defined by the generators

$$T_K^+ = K_q^+ \frac{(K_q^3 - k + 1)}{[K_q^3 - k + 1]} \qquad \text{and} \qquad T_K^- = \frac{(K^3 - k + 1)}{[K_q^3 - k + 1]} K_q^- \tag{11}$$

in a manner similar to the previous cases

$$|\xi)_q = \mathrm{e}_q^{\xi K_q^+} |k; 0\rangle = \mathrm{e}^{\xi T_K^+} |k; 0\rangle = \sum_{n=0}^{\infty} \frac{[2k + n + 1]!}{[n]! [2k + 1]!} \xi^n |k; n\rangle \tag{12}$$

where $\xi \in D^k = \{|\xi|^2 < q^{k-1}\}$. With the normalization factor obtained from the overlap of two states

$$(1-|\xi|^2)_q^{-2k} \equiv {}_q(\xi|\xi)_q = \sum_{n=0}^{\infty} \frac{[2k+n+1]!}{[n]![2k+1]!}|\xi|^{2n} \tag{13}$$

the normalized states are complete

$$1 = \int |\xi)_q \, \mathrm{d}u_q(\xi)_q \, \langle \xi| \qquad \text{with} \qquad \mathrm{d}u_q(\xi) = \frac{[2k-1]}{{}_q(\xi|\xi)_q^{-2}} \mathrm{d}_q^2 \xi$$

and obey the equations

$$(K_q^- + (q^k + q^{-k})\xi[K_q^3] + \xi^2 K_q^+)|\xi)_q = 0$$

and

$$K_q^{\pm}|\xi\rangle_q = \xi^{\mp 1}[K_q^3 \mp k]|\xi\rangle_q .$$

We now proceed with the q-CS propagator utilizing the completeness relations of the q-CS. Let $A = \alpha, z, \xi$, the transition amplitude between coherent states takes the form

$$K = \langle A''|\mathrm{U}(t'', t')|A'\rangle = \int \mathcal{D}\mu_q(A) \exp\left[\sum_{\ell=1}^{L} \ell n \langle A_\ell|A_{\ell-1}\rangle - \frac{\mathrm{i}}{\hbar}\varepsilon \frac{\langle A_\ell|H|A_{\ell-1}\rangle}{\langle A_\ell|A_{\ell-1}\rangle}\right] \tag{14}$$

where

$$\mathcal{D}\mu_q(A) = \lim_{\substack{L\to\infty \\ \varepsilon\to 0}} \prod_{\ell=1}^{L-1} \mathrm{d}\mu_q(A_\ell)$$

and $\varepsilon = (t'' - t')/L$, while H stands for the Hamiltonian the explicit form of which is not important. In the classical limit, considered here as the case where $\hbar \ll$ action, while the deformation parameter q is held fixed, assuming that $A_{\ell-1} \cong A_\ell - \Delta A_\ell$, then from the definition of the CS and the short-time approximation it follows that

$$\varepsilon \cdot \frac{1}{\varepsilon} \ell n \langle A_\ell|A_{\ell-1}\rangle \cong \frac{\varepsilon}{2}\left(\frac{\Delta \bar{A}_\ell}{\varepsilon}\langle A_\ell|T_i^-|A_\ell\rangle - \frac{\Delta A_\ell}{\varepsilon}\langle A_\ell|T_i^+|A_\ell\rangle\right) \tag{15}$$

where $i = a, J, K$ and the bar denotes complex conjugation. In the limit where $L \to \infty$ and $\varepsilon \to 0$ the RHS of the above expression is written formally as $\frac{1}{2}(\dot{\bar{A}}\ \langle A|T_i^+|A\rangle - \dot{A}\langle A|T_i^-|A\rangle)\mathrm{d}t$, where the overdot denotes the time derivative. In this limit

$$K = \int \mathcal{D}\mu_q(A) \exp\left[\frac{\mathrm{i}}{\hbar}\int_0^{t''-t'} \mathrm{d}t\, L(A, \bar{A}; \dot{A}\ \dot{\bar{A}})\right]$$

and the Langrangian is given by

$$L = \frac{\mathrm{i}\hbar}{2}[\dot{A}\langle A|T_i^+|A\rangle - \dot{\bar{A}}\ \langle A|T_i^-|A\rangle] - \mathcal{H}(A, \bar{A}) \tag{16}$$

where $\mathcal{H} = \langle A|H|A\rangle$, from which we extract the canonical 1-form

$$Q = \frac{i\hbar}{2}(\langle A|T_i^+|A\rangle \mathrm{d}A - \langle A|T_i^-|A\rangle \mathrm{d}\bar{A}). \tag{17}$$

By using the properties of the q-CS as above we obtain for the three cases ($\langle\cdot\rangle \equiv \langle A|\cdot|A\rangle$)

$$L = \frac{i\hbar}{2}\left\langle\frac{N+1}{[N+1]}\right\rangle(\dot{\alpha}\bar{\alpha} - \dot{\bar{\alpha}}\,\alpha) - \mathcal{H}(\alpha, \bar{\alpha}) \tag{18a}$$

$$L = \frac{i\hbar}{2}\langle J_q^3 + j\rangle(\dot{z}z^{-1} - \dot{\bar{z}}\,\bar{z}^{-1}) - \mathcal{H}(z, \bar{z}) \tag{18b}$$

and

$$L = \frac{i\hbar}{2}\langle K_q^3 - k\rangle(\dot{\xi}\xi^{-1} - \dot{\bar{\xi}}\,\bar{\xi}^{-1}) - \mathcal{H}(\xi, \bar{\xi}). \tag{18c}$$

Explicitly evaluating the expectation values in the Langrangians, we find that they are modified with respect to their $q = 1$ value [16–19], due to the q-deformation. Let us further recall the fact that in the non-deformed cases α, z and ξ are the coordinates (for another attempt to define path integration with non-commuting coordinates see [20]) of the respective cosets (generalized phase spaces): $\mathrm{WH/U(1)} \approx \mathbb{R}^2$, $\mathrm{SU(2)/U(1)} \approx \mathrm{S}^1 \approx \mathrm{CP}^1$ and $\mathrm{SU(1,1)/U(1)} \approx \mathrm{S}^{1,1} \approx \mathrm{CP}^{1,1}$ and that these spaces are Kähler manifolds with respective potentials $\Phi = \ell n(\alpha|\alpha)$, $(1/2j)\ell n(z|z)$ and $(1/2k)\ell n(\xi|\xi)$, where the states are the usual coherent states of these groups and j and k are the Casimir and Bargmann indices labelling the representations. This potential provides, after exterior differentiation, the canonical 1-form and the invariant metric $\mathrm{d}s^2 = \partial_A\partial_{\bar{A}}\Phi \mathrm{d}A\mathrm{d}\bar{A}$ together with the symplectic 2-form $\omega = \frac{i}{2}\partial_A\partial_{\bar{A}}\Phi \mathrm{d}A \wedge \mathrm{d}\bar{A}$, where $\partial_A \equiv \partial/\partial_A$ [8]. In the presence of deformation, however, the involvement of q-CS, changes the Kähler potential, which in turn changes the metric distance and the symplectic structure of the phase space. We find that

$$\mathrm{d}s^2 = (\langle T_i^- T_i^+\rangle - \langle T_i^-\rangle\langle T_i^+\rangle)\mathrm{d}A\mathrm{d}\bar{A} \tag{19a}$$

and

$$\omega_i = \frac{i}{2}(\langle T_i^- T_i^+\rangle - \langle T_i^-\rangle\langle T_i^+\rangle)\mathrm{d}A \wedge \mathrm{d}\bar{A}. \tag{19b}$$

The explicit evaluation of the metrics for an arbitrary q deformation parameter will be postponed until a future study and here we will continue by observing that on physical grounds one would expect that the physical mechanism of deformation, albeit elusive at present, has quantitatively a rather perturbative character on the non-deformed models to which it would apply.

Reasoning in this way we will proceed by expanding all the operators and states involved in powers of γ ($q = \mathrm{e}^\gamma$) and keep only the first-order terms.

Using the expansion

$$[x] = x + \frac{\gamma^2}{6}(x - x^3) + \frac{\gamma^4}{360}(7x - 10x^3 + 3x^5) + \cdots$$

and the deforming maps connecting the deformed generators of our algebras with their non-deformed counterparts (see for example [21]) we can express the CS generating operators in power series of the deformation parameter as follows

$$T_a^+ = a^+ + \frac{\gamma^2}{12}\{(1-N^2)a^+\} + \cdots \tag{20a}$$

$$T_J^+ = J^+ - \frac{\gamma^2}{12}\{(2J^3 + 2j - 1)J^+\} + \cdots \tag{20b}$$

and

$$T_K^+ = K^+ + \frac{\gamma^2}{12}\{(2K^3 + 2k + 1)K^+\} + \cdots \tag{20c}$$

where a^+, J^+ and K^+ etc, without a q subscript are the ordinary step operators of the respective algebras.

Next we employ the Backer–Champbell–Hausdorff (BCH) formula (see, for example, [22])

$$\exp(A+B) = \exp A \exp B \exp C_2 \exp C_3 \cdots$$

with

$$\begin{aligned} C_2 &= -\tfrac{1}{2}[A, B] \\ C_3 &= \tfrac{1}{6}[A,[A,B]] + \tfrac{1}{3}[B,[A,B]] \qquad \text{etc.} \end{aligned} \tag{21}$$

which is valid for any two non-commutative operators, to provide to first order the following relations between (un-normalized) q-CS, $|\alpha)_q$, $|z)_q$, $|\xi)_q$ and their respective non-deformed $|\alpha)$, $|z)$ and $|\xi)$:

$$\begin{aligned} |\alpha)_q \cong |\alpha) - \frac{\gamma^2}{12}\{&\alpha a^+(N-\alpha a^+)(N-\alpha a^+ + 2) \\ &+ \tfrac{1}{2}\alpha^2 a^{+2}(2N - 2\alpha a^+ + 3) + \tfrac{1}{3}a^3 a^{+3}\}|\alpha) \end{aligned} \tag{22a}$$

$$|z)_q \cong |z) - \frac{\gamma^2}{12}\{2zJ^3J^+ - z^2J^{+2} + (2j-1)zJ^+\}|z) \tag{22b}$$

and

$$|\xi)_q \cong |\xi) - \frac{\gamma^2}{12}\{2\xi K^3K^+ + \xi^2K^{+2} + (2k+1)\xi K^+\}|\xi). \tag{22c}$$

Utilizing these expansions we can now calculate the Kähler potentials Φ_q and the metrics for each case. The symplectic 2-forms to first order in the parameter γ are, for the three algebras of our study, as follows

$$\omega_a = \left\{1 - \frac{\gamma^2}{2}|\alpha|^2(|\alpha|^2 + 2)\right\} \mathrm{d}\bar{\alpha} \wedge \mathrm{d}\alpha \tag{23a}$$

and

$$\omega_\ell = \mathrm{i}\ell p^{-2} - \frac{\gamma^2}{6}\mathrm{i}\ell\bigg\{2\ell p^{-1}(\mp 14\ell - 4)\tau p^{-2} + ((8\ell \pm 3)\tau^2 + (\pm 12\ell + 6)\tau)p^{-3}$$
$$+ (\pm 2\ell + 1)(\mp 10\tau^2 - 5\tau \mp 1)p^{-4}\bigg\}\,\mathrm{d}\bar{\theta}\wedge\mathrm{d}\theta \tag{23b}$$

where the upper sign corresponds to the $SU_q(2)$ case and the lower sign to the $SU_q(1,1)$, and correspondingly $\ell = j$ or k and $\tau = |z|^2$ or $|\xi|^2$ and $p = 1 + |z|^2$ or $1 - |\xi|^2$ when $\theta = z$ or ξ. Similarly the distance metric can be read from the above formulae by dropping the wedge products in the RHS. As was mentioned at the beginning of this letter the deformation manifests itself geometrically in the cosets of the groups, and moreover we now also note that the γ^2 proportional terms are dependent on the modulo of the projective coordinates, which implies an invariance under phase changes of the complex coordinates, for the additional terms in the metrics induced by the deformation.

As a further probe into the geometrical effects of the deformation we will take up the evaluation of the curvature scalar which for our Riemann metric derived from the Φ_q Kähler potential given by the overlap of q-CS, is taken to be of the form

$$R = -(\partial_A\partial_{\bar{A}}\Phi_q)^{-1}\partial_A\partial_{\bar{A}}(\ell n\,\partial_A\partial_{\bar{A}}\Phi_q).$$

For the case of the q-oscillator we find

$$R = \gamma^2 12(1 + 2|\alpha|^2)$$

and similar results hold for the other two cases. Obviously there is a non-zero position dependent curvature with rotational symmetry which tends to zero approaching the zero deformation limit.

Before closing some remarks are in order; the way the Kähler potentials and the Riemannian and symplectic metrics derivable from them, were introduced above, was all by analogy with the non-deformed case. In the non-deformed case however these metrics are invariant, and covariant, correspondingly under the respective canonical transformations associated with each algebra. These same canonical transformations will not however possess the right covariance properties when applied to the metrics derived above, due to the modifications of the latter by extra deformation terms (considering first-order deformation changes for simplicity). Generalized canonical transformations appropriate for the above deformed metrics are therefore required and we hope to take up this problem elsewhere.

In conclusion, a geometrical understanding of the q-deformation of the oscillator, SU(2) and SU(1, 1) algebras has be investigated here using the q-CS path integrals. It is interesting to note that the association of curvature with non-co-commutation at the quantum group level has been discussed before [23] and could probably be related to the curvature found here at the quantum algebra level, by the existing duality between quantum algebras and groups as Hopf algebras. Potential applications of the present results would include the use of quantum groups in addressing quantum mechanical problems in spaces of non-constant curvature, and some problems of this kind are now under study.

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